Limitations of the Van Kampen approach for stochastic epidemiological model

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R_0 and related concepts

Problematic

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Aim

Evaluate and quantify the invasion/extinction probability.

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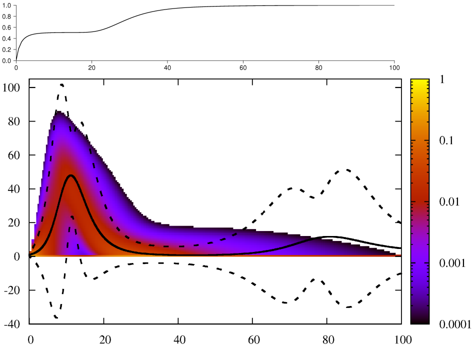
- Van Kampen Expansion: macroscopic comportment and confidence interval for the fluctuations.
- Comparison with numerical integration via EXPOKIT

$$\begin{cases} \frac{dP}{dt} = AP \\ P(t=0) = P_0 \end{cases} \Rightarrow P(t) = e^{tA}P_0$$

Limits of Van Kampen Expansion

When the extinction probability $P(I = 0)$ increases:



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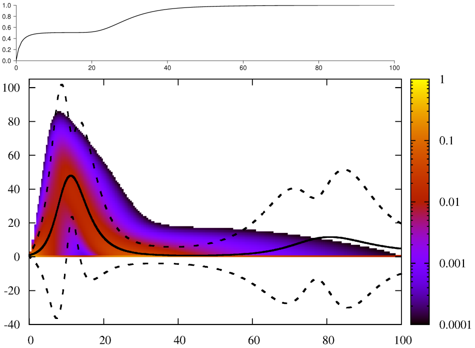

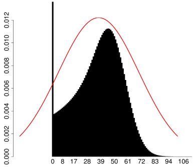

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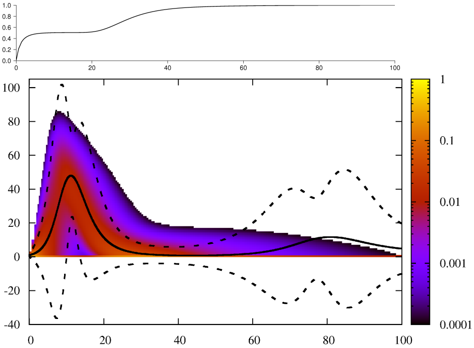

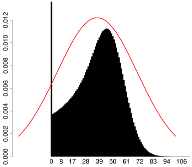

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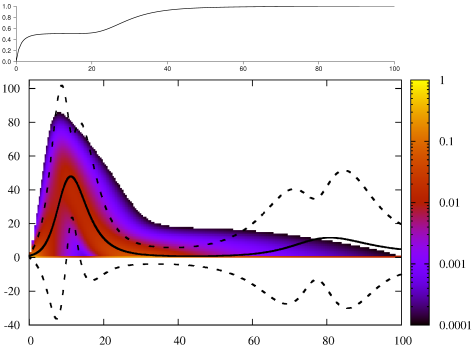
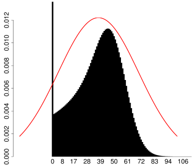


- 

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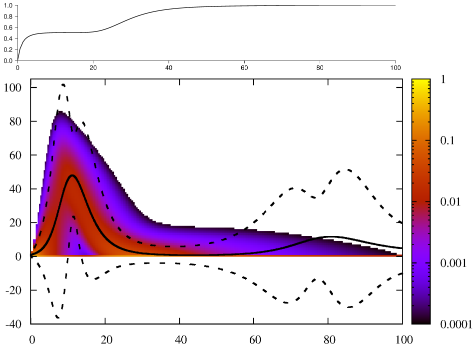

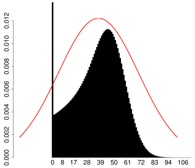

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But qualitative information: distorsions $\Rightarrow$ extinction risk